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Simulations of fluid displacement in heterogeneous porous media

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Abstract. Diffusion-limited aggregation is a proven method for simulation of certain types of two-fluid displacements in porous media. Heterogeneity in permeability can be simulated by setting the lattice spacing equal to the local permeability.

1. Introduction

The phenomenon of viscous fingering has proved to be a formidable barrier to both the economic recovery of oil and attempts to understand the phenomenon itself. A relatively modern review of fingering in porous media has been presented by Stalkup (1983). Recently, however, a new approach has been receiving consideration: an approach based on diffusion-limited aggregation (DLA).

Since the original application of Witten and Sander's (1983) DLA algorithm to flow in porous media (Paterson 1984), a number of workers have developed the concept to include more general problems. For instance, displacements involving arbitrary mobility ratios have been considered (DeGregoria 1985, Sherwood and Nittmann 1986). Attention has also been devoted to surface tension and flow in Hele Shaw cells (Nittmann *et al* 1985, Kadanoff 1985, Tang 1985, Liang 1986).

The DLA model is only an exact model for fluid displacement when there is an exponential distribution of fluid capacities (Chan *et al* 1986), but for many systems DLA may be a useful approximation. This has been demonstrated experimentally by Chen and Wilkinson (1985), Lenormand and Zarcone (1985) and Maloy *et al* (1985).

Although it has been briefly mentioned (Paterson 1984), the algorithm for simulating flow in media with macroscopic permeability variations has not been described in detail. In this paper the algorithm for modelling heterogeneity is described. Some examples of simulations are included. Attention here is focused on the problem of oil recovery from a pattern of wells known as one quarter of a five-spot. It is assumed that the permeable strata is not very thick and that two-dimensional simulations are adequate. However the extension to three dimensions is straightforward. Only simulations at infinite and zero mobility ratios are presented in this paper, although the problem of modelling displacements with an arbitrary mobility ratio is discussed.

2. The DLA analogy

For most situations of practical interest, flow in porous media is governed by Darcy's

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law

$$q = -(k/\mu)\nabla p \tag{1}$$

where q is the volume flow rate per unit area, k is permeability, μ is viscosity and p is pressure. k/μ is known as the mobility. For an incompressible fluid

$$\nabla \cdot q = 0. \tag{2}$$

Thus, for incompressible fluids,

$$\nabla^2 p = 0. \tag{3}$$

The DLA algorithm considers a flux of walkers from a source to a sink (the aggregate) on which the walkers stick. For a steady flux of walkers Witten and Sander (1983) determined the continuum approximation

$$\nabla^2 u = 0 \tag{4}$$

where u is the probability density of finding a walker at a point at any given time. When walkers stick on contact with the aggregate, then the aggregate advances according to

$$v = K\nabla u \tag{5}$$

where $u = 0$ on the boundary of the aggregate. The analogy between diffusion-limited aggregation and two-fluid displacements in porous media is based on the similarity of equations (1) and (3) with (4) and (5). Furthermore, the lattice spacing is a component of K. Thus it is apparent how one might relate K to k/μ. K is also proportional to the sticking probability. Generally the area of a walker upon sticking has been taken as the square of the step length, but this need not be the case.

The anti-DLA algorithm is identical to the DLA algorithm, except that walkers are allowed to step onto the aggregate and eat a piece out of the aggregate rather than sticking to it.

3. Heterogeneous media

We begin by considering the flow of a fluid in porous medium from region 1 of macroscopic permeability k_1 into region 2 of macroscopic permeability k_2. At the permeability discontinuity the normal component of flow rate per unit area must be continuous across the boundary

$$q_1 \cdot n = q_2 \cdot n \tag{6}$$

where n is normal to the boundary and q_1 and q_2 are measured just inside regions 1 and 2 respectively. Additionally, pressure must be continuous along the boundary (Bear 1972)

$$p_1 = p_2. \tag{7}$$

These continuity conditions are equivalent to having a change in lattice cell size in a DLA simulation. At a discontinuity in lattice cell size

$$\nabla u_1 \cdot n = \nabla u_2 \cdot n. \tag{8}$$

With u, the probability of finding a walker at a site, playing the role of pressure

$$u_1 = u_2 \tag{9}$$

in the limit $\Delta x \to 0$, where Δx is the lattice spacing.

This concept is illustrated in one dimension (figure 1) when we consider that the probability of finding a walker at the discontinuity site is $u_i = (u_{i+1} + u_{i-1})/2$. Then

$$\partial u_1/\partial x = (u_i - u_{i-1})/\Delta x_1 \qquad \partial u_2/\partial x = (u_{i+1} - u_i)/\Delta x_2. \tag{10}$$

Conditions (6) and (7) hold when the mobility ratio $(k_1/\mu_1)/(k_2/\mu_2) = \Delta x_1/\Delta x_2$. If the permeability discontinuity occurs within one of the fluids, then $\mu_1 = \mu_2$ and thus $k_1/k_2 = \Delta x_1/\Delta x_2$.

In addition to the conditions at the permeability discontinuity, it is also necessary to consider the condition at the moving interface between the fluids: equation (1). Thus the velocity of the interface has to be proportional to the length of the cells of the lattice. However, the velocity of the interface also depends on the area of the cells, so an adjustment needs to be made. This is perhaps best considered by examining anti-DLA in a thin strip (figure 2). If the two interfaces were some equal distance from the source, walkers will hit the right interface with twice the frequency of the left interface, because twice as many 'left' steps are required to hit the left boundary. Thus the right interface moves twice as fast, as required. However, if the area of the walkers that stick on the left is only one quarter of the area of the walkers that stick on the right, then the right interface will move an additional four times as fast due to this effect. This can be compensated for by allowing only one out of every four walkers that hit the right interface to stick, the rest being forgotten (the method used in this paper). Alternatively, the area of the walkers could be kept constant, with only the step size depending on the lattice spacing. In this manner, the minimum size for a finger could be scaled on some other physical grounds, for example the pore size (see Paterson 1985), or the scaling factor might be the length below which detail is eliminated by hydrodynamic dispersion. Yet another way to adjust the velocity at the interface is to change the number of hits of random walkers to a site before it is occupied, i.e. the number M described by Tang (1985) and Liang (1986), or the number s described by Nittmann and Stanley (1986). However, this also alters the noise experienced by the interface (Nittmann and Stanley 1986). Thus one would need to consider whether the level of noise should be the same in each region of macroscopic permeability.

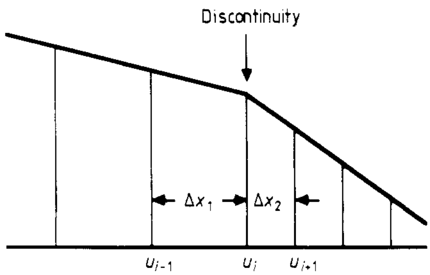

Figure 1. Conditions at a permeability discontinuity in one dimension.

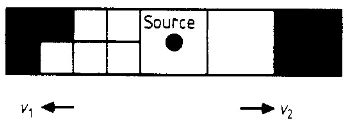

Figure 2. Anti-DLA in a thin strip; consideration of the conditions at the moving interfaces.

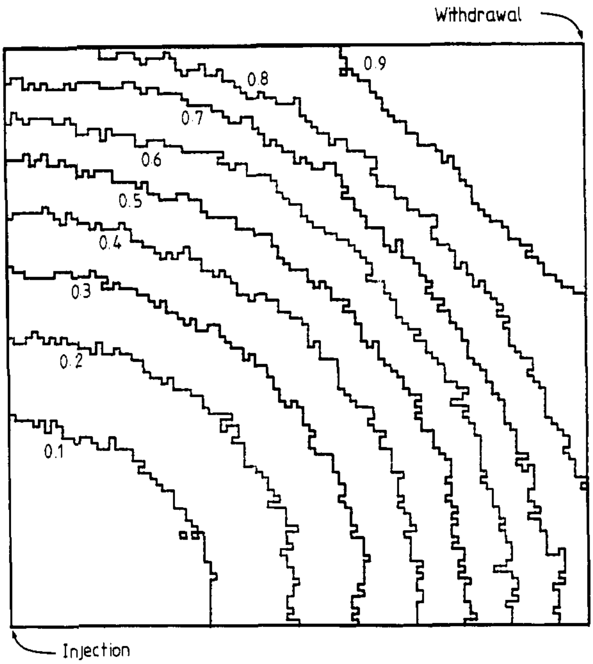

Figure 3. An example of a stable anti-DLA simulation in a region of homogeneous permeability. The region corresponds to a pattern encountered in oil recovery known as one quarter of a five-spot. The numerical values specify the fraction of the total area displaced.

Following an example of an anti-DLA displacement in a region of homogeneous permeability (figure 3), some examples of simulations with regions of different permeabilities are included for both DLA and anti-DLA. Results of simulations with an inner region that is half the permeability of the outer region are shown in figures 4, 5 and 6. The inner region is 60×60 of the larger lattice units, whereas the outer region is bounded by 100×100 lattice units. In figure 4 it is apparent that the low permeability region acts like a lens refracting the interface. Figures 5 and 6 differ in the sequence of random numbers that was used to generate them. In figure 5 the main finger penetrated through the low pemeability region (note again the refraction at the permeability discontinuities) whereas in figure 6 the main finger evaded the low permeability region. Boundary conditions for these examples consists of reflecting the walkers at the limit of the lattice, equivalent to a no-flow condition in an experiment.

4. Mobility ratio

Although this problem has been solved with the inclusion of more conventional numerical methods (DeGregoria 1985, Sherwood and Nittmann 1986), it would be advantageous to be able to simulate using random walkers because of the prospect of better computational speed (Sherwood and Nittmann's simulations took many hours on a moderately sized machine, whereas comparable DLA simulations can be run within an hour or two on a personal computer).

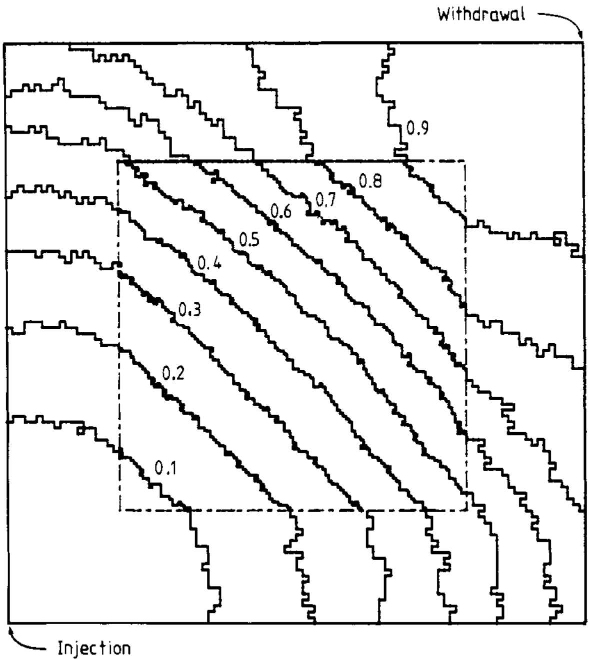

Figure 4. An example of a stable anti-DLA simulation with an inner region that is half the permeability of the outer region.

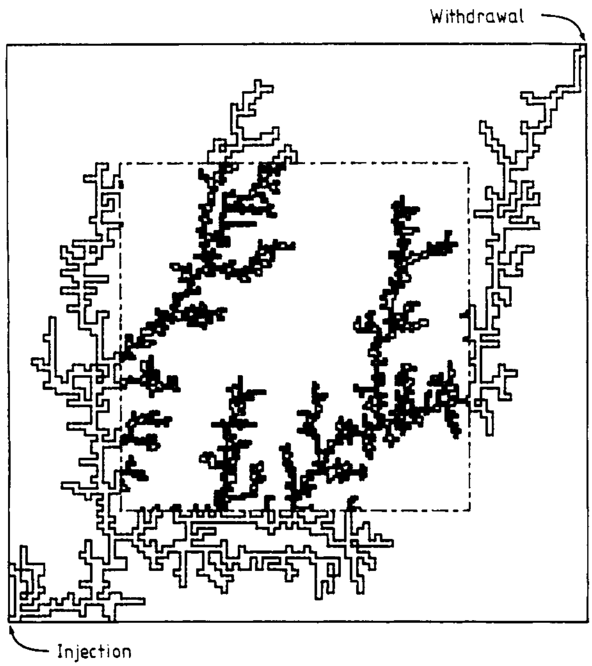

Figure 5. An example of an unstable DLA simulation with an inner region that is half the permeability of the outer region.

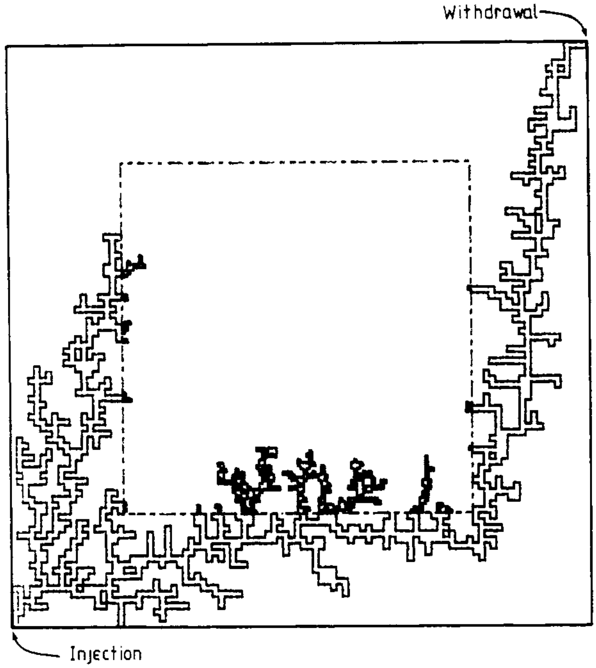

Figure 6. A simulation identical to that used for figure 5, except that a different sequence of random numbers was used.

If the change in lattice size were to move with the moving interface, then we would *almost* have an algorithm for arbitrary mobility ratio. At the interface

$$\boldsymbol{q}_1 \cdot \boldsymbol{n} = \boldsymbol{q}_2 \cdot \boldsymbol{n}. \tag{11}$$

Additionally, pressure must be continuous along the interface:

$$p_1 = p_2. \tag{12}$$

These are identical to the conditions at the permeability discontinuity. The ratio k/μ in Darcy's law shows that k and μ play similar roles. The difference is that permeability k remains fixed with the porous medium, whereas viscosity μ moves with the fluids. Thus the algorithm for arbitrary mobility ratio would release a walker from the injection point and allow it to walk until it reaches the withdrawal point. At the point where the walker crossed the interface the interface would be advanced one unit. The problem is that, because there is an unbiased random walk, a frequent occurrence will be for the walkers to cross the interface many times. At which crossing should the interface be advanced? Until a rational decision can be made, this remains an impasse. An alternative algorithm for arbitrary mobility ratio that utilises random walkers has been proposed (Sahimi and Yortsos 1985). However, this algorithm only considers some of the conditions at the interface and is not expected to be generally applicable.

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